The production of matter from curvature in a particular linearized high order theory of gravity and the longitudinal response function of interferometers

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March 27, 2007

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Abstract

The strict analogy between scalar-tensor theories of gravity and high order gravity is well known in literature. In this paper it is shown that, from a particular high order gravity theory known in literature, it is possible to produce, in the linearized approch, particles which can be seen like massive scalar modes of gravitational waves and the response of interferometers to this type of particles is analyzed. The presence of the mass generates a longitudinal force in addition of the transverse one which is proper of the massless gravitational waves and the response of an arm of an interferometer to this longitudinal effect in the frame of a local observer is computed. This longitudinal response function is directly connected with the function of the Ricci scalar in the particular action of this high order theory. Important conseguences from a theoretical point of view could arise from this approach, because it opens to the possibility of using the signals seen from interferometers to understand which is the correct theory of gravitation.

The presence of the mass could also have important applications in cosmology because the fact that gravitational waves can have mass could give a contribution to the dark matter of the Universe.

PACS numbers: 04.80.Nn, 04.30.Nk, 04.50.+h

1 Introduction

The design and construction of a number of sensitive detectors for gravitational waves is underway today. There are some laser interferometers like the Virgo detector, being built in Cascina, near Pisa, Italy, by a joint Italian-French collaboration, the GEO 600 detector being built in Hannover, Germany, by a joint Anglo-Germany collaboration, the two LIGO detectors being built in the United States (one in Hanford, Washington and the other in Livingston, Louisiana) by a joint Caltech-Mit collaboration, and the TAMA 300 detector, being built near Tokyo, Japan. Many bar detectors are currently in operation too, and several interferometers and bars are in a phase of planning and proposal stages (for the current status of gravitational waves experiments see [1, 2]).

The results of these detectors will have a fundamental impact on astrophysics and gravitation physics. There will be lots of experimental data to be analyzed, and theorists will be forced to interact with lots of experiments and data analysts to extract the physics from the data stream.

In this paper the production and the potential detection with interferometers of a hypotetical massive *scalar* component of gravitational radiation which arises from a particular high order theory of gravity well known in literature (see [3]) is shown. This agrees with the formal equivalence between high order theories of gravity and scalar tensor gravity which is well known in literature [3, 4, 5, 6].

In the second Section of this paper it is shown that a massive scalar mode of gravitational radiation arises from the high order action [3]

$$S = \int d^4x\sqrt{-g}R^{-1} + \mathcal{L}_m, \tag{1}$$

where R is the Ricci scalar curvature. Equation (1) is a particular choice with respect the well known canonical one of General Relativity (the Einstein - Hilbert action [7, 8]) which is

$$S = \int d^4x\sqrt{-g}R + \mathcal{L}_m. \tag{2}$$

The presence of the mass could also have important applications in cosmology because the fact that gravitational waves can have mass could give a contribution to the dark matter of the Universe. We also recall that an alternative way to resolve the dark matter and dark energy problems using high order gravity is shown in ref. [9].

In Section three it is shown that this massive component generates a longitudinal force in addition of the transverse one which is proper of the massless case.

After this, in Section four, the potential interferometric detection of this longitudinal component is analyzed and the response of an interferometer is computed. It is also shown that this longitudinal response function is directly connected with the Ricci scalar R. This connection opens to the possibility of using the signals seen from interferometers to understand which is the correct theory of gravitation [6, 10].

In the analysis of the longitudinal response of interferometers a computation parallel to the one seen in [6] and [11] will be used.

2 The production of a scalar massive mode of gravitational radiation in the R^{-1} high order theory of gravity

If the gravitational Lagrangian is nonlinear in the curvature invariants the Einstein field equations has an order higher than second [4, 6]. For this reason such theories are often called higher-order gravitational theories. This is exactly the case of the action (1).

By varying this action with respect to $g_{\mu\nu}$ (see refs. [4, 6] for a parallel computation) the field equations are obtained (note in this paper we work with $G=1$, $c=1$ and $\hbar=1$):

$$\begin{array}{c} G_{\mu\nu}=4\pi\tilde{G}R^{2}T_{\mu\nu}^{(m)}-\frac{R^{5}}{4}((R^{-2})_{;\mu}(R^{-2})_{;\nu}-\frac{1}{2}g_{\mu\nu}g^{\alpha\beta}(R^{-2})_{;\alpha}(R^{-2})_{;\beta})+\\ +R^{2}((R^{-2})_{;\mu\nu}-g_{\mu\nu}[]R^{-2})-\frac{R^{2}}{2}g_{\mu\nu}g^{\mu\nu}(R^{-2})_{;\mu}(R^{-2})_{;\nu} \end{array} \tag{3}$$

with associed a Klein - Gordon equation for the R^{-2} scalar field

$$\begin{array}{c} []R^{-2}=\frac{-2}{-R+6}[-4\pi\tilde{G}T^{(m)}+g^{\mu\nu}(R^{-2})_{;\mu}(R^{-2})_{;\nu}+\\ -R^{-2}\frac{d}{d(R^{-2})}(\frac{1}{2}g^{\mu\nu}(R^{-2})_{;\mu}(R^{-2})_{;\nu})+\frac{1}{4}\frac{dR}{d(R^{-2})}g^{\mu\nu}(R^{-2})_{;\mu}(R^{-2})_{;\nu}], \end{array} \tag{4}$$

where $[]$ is the Dalembertian operator.

In the above equations $T_{\mu\nu}^{(m)}$ is the ordinary stress-energy tensor of the matter and $\tilde{G}$ is a dimensional, strictly positive, constant [4, 6]. The Newton constant is replaced by the effective coupling

$$G_{eff}=-\frac{R^{2}}{2}, \tag{5}$$

which is different from G.

To study gravitational waves the linearized theory in vacuum ($T_{\mu\nu}^{(m)}=0$) has to be analyzed, with a little perturbation of the background, which is assumed given by the Minkowskian background plus $R^{-2}=(R^{-2})_{0}$ (i.e., the Ricci scalar is assumed constant).

In the linearized theory it is also

$$\frac{1}{2}g^{\mu\nu}(R^{-2})_{;\mu}(R^{-2})_{;\nu}\simeq\frac{1}{2}\eta^{\mu\nu}(R^{-2})_{;\mu}(R^{-2})_{;\nu} \tag{6}$$

where $\eta^{\mu\nu}$ is the flat metric tensor of the Minkowskian background. Thus $\frac{1}{2}g^{\mu\nu}(R^{-2})_{;\mu}(R^{-2})_{;\nu}$ is proportional to the square of the total covariant derivative in good approximation. It can also be written like

$$\frac{1}{2}g^{\mu\nu}(R^{-2})_{;\mu}(R^{-2})_{;\nu}\simeq\beta\delta(-R^{-2})^{2}\Rightarrow$$
$$\Rightarrow\frac{d}{d(R^{-2})}(\frac{1}{2}g^{\mu\nu}(R^{-2})_{;\mu}(R^{-2})_{;\nu})\simeq2\beta\delta(-R^{-2}). \tag{7}$$

Putting

$$g_{\mu\nu}=\eta_{\mu\nu}+h_{\mu\nu}$$
$$(-R^{-2})_{*}=-(R^{-2})_{0}+\delta(-R^{-2}), \tag{8}$$

to first order in $h_{\mu\nu}$ and $\delta(-R^{-2})$, calling $\widetilde{R}_{\mu\nu\rho\sigma}$, $\widetilde{R}_{\mu\nu}$ and $\widetilde{R}$ the linearized quantity which correspond to $R_{\mu\nu\rho\sigma}$, $R_{\mu\nu}$ and R, the linearized field equations are obtained [6, 8]:

$$\widetilde{R}_{\mu\nu}-\frac{\widetilde{R}}{2}\eta_{\mu\nu}=\partial_{\mu}\partial_{\nu}\xi-\eta_{\mu\nu}[]\xi$$
$$[]\xi=-m^{2}\xi, \tag{9}$$

where

$$\xi\equiv\frac{\delta(-R^{-2})}{(-R^{-2})_{0}}$$
$$m^{2}\equiv\frac{-\beta(\frac{R}{4})_{0}}{-2(\frac{R}{4})_{0}+3}=\frac{\beta R_{0}}{2R_{0}-12}. \tag{10}$$

We emphasize that the mass is directly generated by a function of the Ricci scalar (i.e. by curvature).

$\widetilde{R}_{\mu\nu\rho\sigma}$ and eqs. (9) are invariants for gauge transformations [6, 8]

$$h_{\mu\nu}\rightarrow h'_{\mu\nu}=h_{\mu\nu}-\partial_{(\mu}\epsilon_{\nu)}$$
$$\delta(R^{-2})\rightarrow\delta(R^{-2})'=\delta(R^{-2}); \tag{11}$$

then

$$\bar{h}_{\mu\nu}\equiv h_{\mu\nu}-\frac{h}{2}\eta_{\mu\nu}-\eta_{\mu\nu}\xi \tag{12}$$

can be defined, and, considering the transform for the parameter ϵ^{μ}

$$[]\epsilon_{\nu}=\partial^{\mu}\bar{h}_{\mu\nu}, \tag{13}$$

a gauge parallel to the Lorenz one of electromagnetic waves can be choosen:

$$\partial^{\mu}\bar{h}_{\mu\nu}=0. \tag{14}$$

In this way field equations read like

$$[]\bar{h}_{\mu\nu}=0 \tag{15}$$

$$\Box\xi = -m^2\xi. \tag{16}$$

Solutions of eqs. (15) and (16) are plan waves:

$$\bar{h}_{\mu\nu} = A_{\mu\nu}(\overrightarrow{p})\exp(ip^{\alpha}x_{\alpha}) + c.c. \tag{17}$$

$$\xi = a(\overrightarrow{p})\exp(iq^{\alpha}x_{\alpha}) + c.c. \tag{18}$$

where

$$\begin{aligned} k^{\alpha} &\equiv (\omega, \overrightarrow{p}) \qquad \omega = p \equiv |\overrightarrow{p}| \\ q^{\alpha} &\equiv (\omega_m, \overrightarrow{p}) \qquad \omega_m = \sqrt{m^2+p^2}. \end{aligned} \tag{19}$$

In eqs. (15) and (17) the equation and the solution for the tensorial waves exactly like in General Relativity [7, 8] have been obteined, while eqs. (16) and (18) are respectively the equation and the solution for the scalar mode (see also [6]).

The fact that the dispersion law for the modes of the scalar massive field ξ is not linear has to be emphatized. The velocity of every tensorial mode $\bar{h}_{\mu\nu}$ is the light speed c, but the dispersion law (the second of eq. (19)) for the modes of ξ is that of a massive field which can be discussed like a wave-packet [6, 12]. Also, the group-velocity of a wave-packet of ξ centered in $\overrightarrow{p}$ is

$$\overrightarrow{v_G} = \frac{\overrightarrow{p}}{\omega}, \tag{20}$$

which is exactly the velocity of a massive particle with mass m and momentum $\overrightarrow{p}$.

From the second of eqs. (19) and eq. (20) it is simple to obtain:

$$v_G = \frac{\sqrt{\omega^2 - m^2}}{\omega}. \tag{21}$$

Then, wanting a constant speed of our wave-packet, it has to be [6]

$$m = \sqrt{(1 - v_G^2)}\omega. \tag{22}$$

The relation (22) is shown in fig. 1 for a value $v_G = 0.9$.

Now the analisys can remain in the Lorenz gauge with trasformations of the type $\Box\epsilon_{\nu} = 0$; this gauge gives a condition of transversality for the tensorial part of the field: $k^{\mu}A_{\mu\nu} = 0$, but does not give the transversality for the total field $h_{\mu\nu}$. From eq. (12) it is

$$h_{\mu\nu} = \bar{h}_{\mu\nu} - \frac{\bar{h}}{2}\eta_{\mu\nu} - \eta_{\mu\nu}\xi. \tag{23}$$

At this point, if being in the massless case [6], it could been put

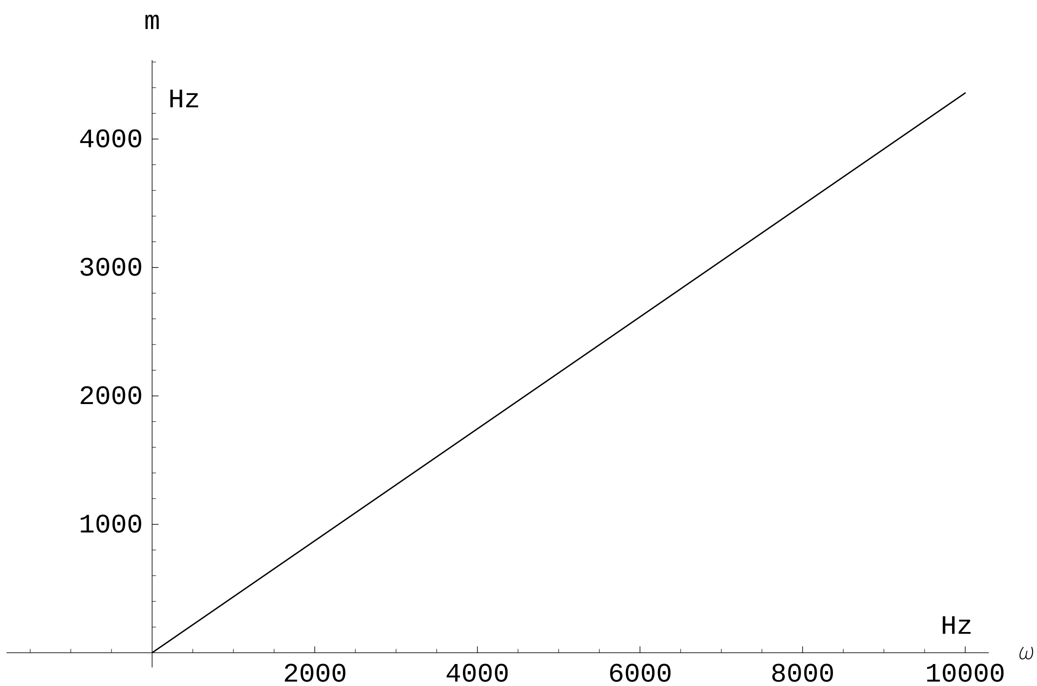

Figure 1: the mass-frequency relation for a massive SGW propagating with a speed of $0.9c$: for the mass it is $1Hz = 10^{-15}eV$

$$\begin{aligned} &\Box\epsilon^{\mu} = 0 \\ &\partial_{\mu}\epsilon^{\mu} = -\tfrac{\bar{h}}{2} - \xi, \end{aligned} \tag{24}$$

which gives the total transversality of the field. But in the massive case this is impossible. In fact, applying the Dalembertian operator to the second of eqs. (24) and using the field equations (15) and (16) it results

$$\Box\epsilon^{\mu} = -m^2\xi, \tag{25}$$

which is in contrast with the first of eqs. (24). In the same way it is possible to show that it does not exist any linear relation between the tensorial field $\bar{h}_{\mu\nu}$ and the scalar field ξ. Thus a gauge in wich $h_{\mu\nu}$ is purely spatial cannot be chosen (i.e. it cannot be put $h_{\mu 0} = 0$, see eq. (23)) . But the traceless condition to the field $\bar{h}_{\mu\nu}$ can be put :

$$\begin{aligned} &\Box\epsilon^{\mu} = 0 \\ &\partial_{\mu}\epsilon^{\mu} = -\tfrac{\bar{h}}{2}. \end{aligned} \tag{26}$$

These equations imply

$$\partial^{\mu}\bar{h}_{\mu\nu} = 0. \tag{27}$$

To save the conditions $\partial_{\mu}\bar{h}^{\mu\nu}$ and $\bar{h} = 0$ transformations like

$$\begin{aligned} &\Box\epsilon^{\mu} = 0 \\ &\partial_{\mu}\epsilon^{\mu} = 0 \end{aligned} \tag{28}$$

can be used and, taking $\overrightarrow{p}$ in the z direction, a gauge in which only A_{11}, A_{22}, and $A_{12} = A_{21}$ are different to zero can be chosen. The condition $\bar{h} = 0$ gives $A_{11} = -A_{22}$. Now, putting these equations in eq. (23) and defining $\Phi \equiv -\xi$ it results

$$h_{\mu\nu}(t,z) = A^{+}(t-z)e_{\mu\nu}^{(+)} + A^{\times}(t-z)e_{\mu\nu}^{(\times)} + \Phi(t - v_G z)\eta_{\mu\nu}. \tag{29}$$

The term $A^{+}(t-z)e_{\mu\nu}^{(+)} + A^{\times}(t-z)e_{\mu\nu}^{(\times)}$ describes the two standard (i.e. tensorial) polarizations of gravitational waves which arise from General Relativity, while the term $\Phi(t - v_G z)\eta_{\mu\nu}$ is the scalar massive field arising from the high order theory.

3 The origin of a longitudinal force

For a purely scalar gravitational wave eq. (29) can be rewritten as

$$h_{\mu\nu}(t - v_G z) = \Phi(t - v_G z)\eta_{\mu\nu} \tag{30}$$

and the corrispondent line element is

$$ds^2 = [1 + \Phi(t - v_G z)](-dt^2 + dz^2 + dx^2 + dy^2). \tag{31}$$

But, in a laboratory environment on Earth, the coordinate system in which the space-time is locally flat [6, 7, 8] is typically used and the distance between any two points is given simply by the difference in their coordinates in the sense of Newtonian physics. This frame is the proper reference frame of a local observer, located for example in the position of the beam splitter of an interferometer. In this frame gravitational waves manifest themself by exerting tidal forces on the masses (the mirror and the beam-splitter in the case of an interferometer). A detailed analysis of the frame of the local observer is given in ref. [8], sect. 13.6. Here only the more important features of this coordinate system are remembered:

the time coordinate x_0 is the proper time of the observer O;

spatial axes are centered in O;

in the special case of zero acceleration and zero rotation the spatial coordinates x_j are the proper distances along the axes and the frame of the local observer reduces to a local Lorentz frame: in this case the line element reads [8]

$$ds^2 = -(dx^0)^2 + \delta_{ij} dx^i dx^j + O(|x^j|^2) dx^\alpha dx^\beta. \tag{32}$$

The effect of the gravitational wave on test masses is described by the equation

$$\ddot{x}^i = -\widetilde{R}^i_{0k0} x^k, \tag{33}$$

which is the equation for geodesic deviation in this frame.

Thus, to study the effect of the scalar gravitational wave on test masses, $\widetilde{R}^i_{0k0}$ has to be computed in the proper reference frame of the local observer. But, because the linearized Riemann tensor $\widetilde{R}_{\mu\nu\rho\sigma}$ is invariant under gauge transformations [6, 8, 12], it can be directly computed from eq. (30).

From [8] it is:

$$\widetilde{R}_{\mu\nu\rho\sigma} = \frac{1}{2}\{\partial_\mu \partial_\beta h_{\alpha\nu} + \partial_\nu \partial_\alpha h_{\mu\beta} - \partial_\alpha \partial_\beta h_{\mu\nu} - \partial_\mu \partial_\nu h_{\alpha\beta}\}, \tag{34}$$

that, in the case eq. (30), begins

$$\widetilde{R}^\alpha_{0\gamma0} = \frac{1}{2}\{\partial^\alpha \partial_0 \Phi \eta_{0\gamma} + \partial_0 \partial_\gamma \Phi \delta^\alpha_0 - \partial^\alpha \partial_\gamma \Phi \eta_{00} - \partial_0 \partial_0 \Phi \delta^\alpha_\gamma\}; \tag{35}$$

the different elements are (only the non zero ones will be written):

$$\partial^\alpha \partial_0 \Phi \eta_{0\gamma} = \left\{ \begin{array}{ccc} \partial_t^2 \Phi & for & \alpha = \gamma = 0 \\ \\ -\partial_z \partial_t \Phi & for & \alpha = 3; \gamma = 0 \end{array} \right\} \tag{36}$$

$$\partial_0 \partial_\gamma \Phi \delta^\alpha_0 = \left\{ \begin{array}{ccc} \partial_t^2 \Phi & for & \alpha = \gamma = 0 \\ \\ \partial_t \partial_z \Phi & for & \alpha = 0; \gamma = 3 \end{array} \right\} \tag{37}$$

$$
-\partial^{\alpha}\partial_{\gamma}\Phi\eta_{00}=\partial^{\alpha}\partial_{\gamma}\Phi=\left\{\begin{array}{ccc} -\partial_t^2\Phi & for & \alpha=\gamma=0 \\ \\ \partial_z^2\Phi & for & \alpha=\gamma=3 \\ \\ -\partial_t\partial_z\Phi & for & \alpha=0;\gamma=3 \\ \\ \partial_z\partial_t\Phi & for & \alpha=3;\gamma=0 \end{array}\right\} \tag{38}
$$

$$
-\partial_0\partial_0\Phi\delta_{\gamma}^{\alpha}= \ -\partial_z^2\Phi \quad for \quad \alpha=\gamma \ . \tag{39}
$$

Now, putting these results in eq. (35) it results:

$$
\begin{array}{c} \widetilde{R}^1_{010}=-\frac{1}{2}\ddot{\Phi} \\ \\ \widetilde{R}^2_{010}=-\frac{1}{2}\ddot{\Phi} \\ \\ \widetilde{R}^3_{030}=\frac{1}{2}\Box\Phi. \end{array} \tag{40}
$$

But, putting the field equation (16) in the third of eqs. (40) it is

$$
\widetilde{R}^3_{030}=\frac{1}{2}m^2\Phi, \tag{41}
$$

which shows that the field is not transversal.
Infact, using eq. (33) it results

$$
\ddot{x}=\frac{1}{2}\ddot{\Phi}x, \tag{42}
$$

$$
\ddot{y}=\frac{1}{2}\ddot{\Phi}y \tag{43}
$$

and

$$
\ddot{z}=-\frac{1}{2}m^2\Phi(t-v_G z)z. \tag{44}
$$

Then the effect of the mass is the generation of a *longitudinal* force (in addition to the transverse one). Note that in the limit $m\rightarrow 0$ the longitudinal force vanishes.

4 Analysis of the interferometer's response to the longitudinal component

Before starting the analysis it has to be discussed if there are fenomenogical limitations to the mass of the scalar particle [12, 13]. Treating scalars like classical waves, that act coherently with the interferometer, it has to be $m \ll 1/L$, where $L=3$ kilometers in the case of Virgo and $L=4$ kilometers in the case of LIGO [1, 2, 6]. Thus it has to be approximately $m < 10^{-9}eV$. However there is a stronger limitation coming from the fact that the scalar wave needs a frequency which falls in the frequency-range for earth based gravitational antennas that is the interval $10Hz \leq f \leq 10KHz$ [1, 2]. For a massive scalar gravitational wave, from the second of eqs. (19) it is:

$$2\pi f = \omega = \sqrt{m^2 + p^2}, \tag{45}$$

were p is the momentum [13]. Thus it needs

$$0eV \leq m \leq 10^{-11}eV. \tag{46}$$

For these light scalars their effect can be still discussed as a coherent gravitational wave.

Equations (42), (43) and (44) give the tidal acceleration of the test mass caused by the scalar gravitational wave respectly in the x direction, in the y direction and in the z direction [6, 11].

Equivalently we can say that there is a gravitational potential [6, 8, 11]:

$$V(\overrightarrow{r}, t) = -\frac{1}{4}\ddot{\Phi}(t - \frac{z}{v_P})[x^2 + y^2] + \frac{1}{2}m^2 \int_0^z \Phi(t - v_G z) a da, \tag{47}$$

which generates the tidal forces, and that the motion of the test mass is governed by the Newtonian equation

$$\ddot{\overrightarrow{r}} = - \bigtriangledown V. \tag{48}$$

To obtain the longitudinal component of the scalar gravitational wave the solution of eq. (44) has to be found.

For this goal the perturbation method can be used. A function of time for a fixed z, $\psi(t - v_G z)$, can be defined [6], for which it is

$$\ddot{\psi}(t - v_G z) \equiv \Phi(t - v_G z) \tag{49}$$

(note: the most general definition is $\psi(t - v_G z) + a(t - v_G z) + b$, but, assuming only small variatons in the positions of the test masses, it results $a = b = 0$).

In this way it results

$$\delta z(t - v_G z) = -\frac{1}{2}m^2 z_0 \psi((t - v_G z). \tag{50}$$

A feature of the frame of a local observer is the coordinate dependence of the tidal forces due by scalar gravitational waves which can be changed with a mere shift of the origin of the coordinate system [6, 11]:

$$x \rightarrow x + x', \;\; y \rightarrow y + y' \text{ and } z \rightarrow z + z'. \tag{51}$$

The same applies to the test mass displacements, in the z direction, eq. (50). This is an indication that the coordinates of a local observer are not simple as they could seem [6, 8, 11].

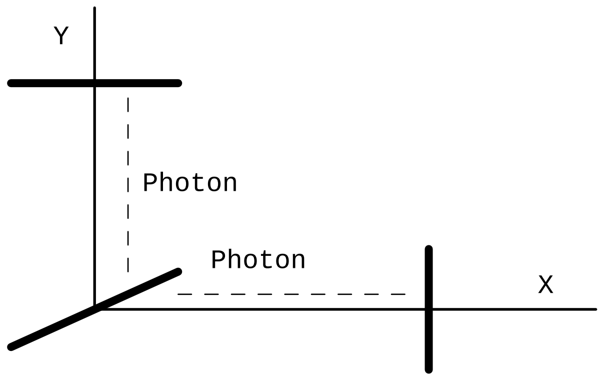

Figure 2: photons can be launched from the beam-splitter to be bounced back by the mirror

Now, let us consider the relative motion of test masses. A good way to analyze variations in the proper distance (time) of test masses is by means of "bouncing photons" (see refs. [6, 11] and figure 2). A photon can be launched from the beam-splitter to be bounced back by the mirror. It will be assumed that both the beam-splitter and the mirror are located along the z axis of our coordinate system (i.e. an arm of the interferometer is in the z direction, which is the direction of the propagating massive scalar gravitational wave and of the longitudinal force, see also Figure 3).

It will be shown that, in the frame of a local observer, two different effects have to be considered in the calculation of the variation of the round-trip time for photons, in analogy with the cases of [6] and [11], where the considered effects were three, but, if we put the beam splitter in the origin of our coordinate system, the third effect vanishes.

The unperturbed coordinates for the beam-splitter and the mirror are $x_b = 0$ and $x_m = L$. So the unperturbed propagation time between the two masses is

$$T = L. \tag{52}$$

From eq. (50) it results that the displacements of the two masses under the influence of the scalar gravitational wave are

$$\delta z_b(t) = 0 \tag{53}$$

and

$$\delta z_m(t - v_G L) = -\frac{1}{2}m^2 L\psi(t - v_G L). \tag{54}$$

In this way, the relative displacement, is

$$\delta L(t) = \delta z_m(t - v_G L) - \delta z_b(t) = -\frac{1}{2}m^2 L\psi(t - v_G L), \tag{55}$$

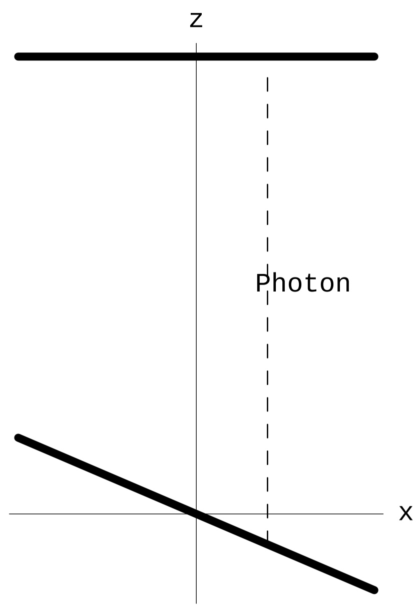

Figure 3: the beam splitter and the mirror are located in the direction of the incoming SGW

Thus it results

$$\frac{\delta L(t)}{L}=\frac{\delta T(t)}{T}=-\frac{1}{2}m^{2}\psi(t-v_{G}L). \tag{56}$$

But there is the problem that, for a large separation between the test masses (in the case of Virgo or LIGO the distance between the beam-splitter and the mirror is three or four kilometers), the definition (55) for relative displacement becomes unphysical because the two test masses are taken at the same time and therefore cannot be in a casual connection [6, 11]. The correct definitions for our bouncing photon can be written like

$$\delta L_{1}(t)=\delta z_{m}(t-v_{G}L)-\delta z_{b}(t-T_{1}) \tag{57}$$

and

$$\delta L_{2}(t)=\delta z_{m}(t-v_{G}L-T_{2})-\delta z_{b}(t), \tag{58}$$

where T_1 and T_2 are the photon propagation times for the forward and return trip correspondingly. According to the new definitions, the displacement of one test mass is compared with the displacement of the other at a later time to allow for finite delay from the light propagation. Note that the propagation times T_1 and T_2 in eqs. (57) and (58) can be replaced with the nominal value T because the test mass displacements are alredy first order in Φ. Thus, for the total change in the distance between the beam splitter and the mirror in one round-trip of the photon, it is

$$\delta L_{r.t.}(t) = \delta L_1(t-T)+\delta L_2(t) = 2\delta z_m(t-v_G L-T)-\delta z_b(t)-\delta z_b(t-2T), \quad (59)$$

and in terms of the amplitude and mass of the SGW:

$$\delta L_{r.t.}(t) = -m^2 L\psi(t - v_G L - T). \quad (60)$$

The change in distance (60) leads to changes in the round-trip time for photons propagating between the beam-splitter and the mirror:

$$\frac{\delta_1 T(t)}{T} = -m^2\psi(t - v_G L - T). \quad (61)$$

In the last calculation (variations in the photon round-trip time which come from the motion of the test masses inducted by the scalar gravitational wave), it was implicitly assumed that the propagation of the photon between the beam-splitter and the mirror of our interferometer is uniform as if it were moving in a flat space-time. But the presence of the tidal forces indicates that the space-time is curved. As a result another effect after the previous has to be considered, which requires spacial separation [6, 11].

For this effect we consider the interval for photons propagating along the z-axis

$$ds^2 = g_{00}dt^2 + dz^2. \quad (62)$$

The condition for a null trajectory ($ds = 0$) gives the coordinate velocity of the photons

$$v^2 \equiv (\frac{dz}{dt})^2 = 1 + 2V(t, z), \quad (63)$$

which to first order in Φ is approximated by

$$v \approx \pm[1 + V(t, z)], \quad (64)$$

with $+$ and $-$ for the forward and return trip respectively. Knowing the coordinate velocity of the photon, the propagation time for its travelling between the beam-splitter and the mirror can be defined:

$$T_1(t) = \int_{z_b(t-T_1)}^{z_m(t)} \frac{dz}{v} \quad (65)$$

and

$$T_2(t) = \int_{z_m(t-T_2)}^{z_b(t)} \frac{(-dz)}{v}. \quad (66)$$

The calculations of these integrals would be complicated because the boundary $z_m(t)$ is changing with time. In fact it is

$$z_b(t) = \delta z_b(t) = 0 \tag{67}$$

but

$$z_m(t) = L + \delta z_m(t). \tag{68}$$

But, to first order in Φ, this contribution can be approximated by $\delta L_2(t)$ (see eq. (58)). Thus, the combined effect of the varying boundary is given by $\delta_1 T(t)$ in eq. (61). Then only the times for photon propagation between the fixed boundaries 0 and L have to be calculated. Such propagation times will be denoted with $\Delta T_{1,2}$ to distinguish from $T_{1,2}$. In the forward trip, the propagation time between the fixed limits is

$$\Delta T_1(t) = \int_0^L \frac{dz}{v(t', z)} \approx T - \int_0^L V(t', z) dz, \tag{69}$$

where t' is the retardation time which corresponds to the unperturbed photon trajectory:

$$t' = t - (L - z)$$

(i.e. t is the time at which the photon arrives in the position L, so $L - z = t - t'$).

Similiary, the propagation time in the return trip is

$$\Delta T_2(t) = T - \int_L^0 V(t', z) dz, \tag{70}$$

where now the retardation time is given by

$$t' = t - z.$$

The sum of $\Delta T_1(t - T)$ and $\Delta T_2(t)$ gives the round-trip time for photons traveling between the fixed boundaries. Then the deviation of this round-trip time (distance) from its unperturbed value $2T$ is

$$\delta_2 T(t) = \int_0^L [V(t - 2T + z, z) + V(t - z, z)] dz. \tag{71}$$

From eqs. (47) and (71) it results:

$$\begin{aligned} \delta_2 T(t) = \tfrac{1}{2} m^2 \textstyle\int_0^L [\int_0^z \Phi(t - 2T + a - v_G a) a da + \int_0^z \Phi(t - a - v_G a) a da] dz = \\ = \tfrac{1}{4} m^2 \textstyle\int_0^L [\Phi(t - v_G z - 2T + z) + \Phi(t - v_G z - z)] z^2 dz + \\ - \tfrac{1}{4} m^2 \textstyle\int_0^L [\int_0^z \Phi'(t - 2T + a - v_G a) z^2 da + \int_0^z \Phi'(t - a - v_G a) z^2 da] dz, \end{aligned} \tag{72}$$

Thus the total round-trip proper distance in presence of the scalar gravitational wave is:

$$T = 2T + \delta_1 T + \delta_2 T. \tag{73}$$

Now, to obtain the interferometer response function of the massive scalar field, the analysis can be transled in the frequency domine.

Using the Fourier transform of ψ defined from

$$\tilde{\psi}(\omega) = \int_{-\infty}^{\infty} dt \psi(t) \exp(i\omega t), \tag{74}$$

eq. (61) can be rewritten like:

$$\frac{\delta_1 \tilde{T}(\omega)}{T} = -m^2 \Upsilon_1^*(\omega) \tilde{\psi}(\omega) \tag{75}$$

with

$$\Upsilon_1^*(\omega) = \exp[i\omega(1 + v_G)L]. \tag{76}$$

But, from a theorem about Fourier transforms, it's simple to obtain:

$$\tilde{\psi}(\omega) = -\frac{\tilde{\Phi}(\omega)}{\omega^2}, \tag{77}$$

where

$$\tilde{\Phi}(\omega) = \int_{-\infty}^{\infty} dt \Phi(t) \exp(i\omega t). \tag{78}$$

is the Fourier transform of our scalar field.

Then it results:

$$\frac{\delta_1 \tilde{T}(\omega)}{T} = \frac{m^2}{\omega^2} \Upsilon_1^*(\omega) \tilde{\Phi}(\omega), \tag{79}$$

and, defining:

$$\Upsilon_1 \equiv \frac{m^2}{\omega^2} \Upsilon_1^*(\omega) = (1 - v_G^2) \Upsilon_1^*(\omega), \tag{80}$$

we obtain:

$$\frac{\delta_1 \tilde{T}(\omega)}{T} = \Upsilon_1(\omega) \tilde{\Phi}(\omega). \tag{81}$$

On the other hand eq. (72) can be rewritten in the frequency space like:

$$\begin{array}{c} \delta_2 \tilde{T}(\omega) = \frac{1}{2\omega(v_G^2 - 1)^2} [\exp[2i\omega L](v_G + 1)^3 (-2i + \omega L(v_G - 1) + \\ + \qquad 2\exp[i\omega L(1 + v_G)](6iv_G + 2iv_G^3 - \omega L + \omega L v_G^4) + \\ +(v_G + 1)^3(-2i + \omega L(v_G + 1))] \tilde{\Phi}(\omega). \end{array} \tag{82}$$

Now

$$\frac{\delta_2\tilde{T}(\omega)}{T}=\Upsilon_2(\omega)\tilde{\Phi}(\omega), \tag{83}$$

can be put, with

$$\begin{array}{c}
\Upsilon_2(\omega)=\frac{1}{2\omega L(v_G^2-1)^2}[\exp[2i\omega L](v_G+1)^3(-2i+\omega L(v_G-1)+\\
2\exp[i\omega L(1+v_G)](6iv_G+2iv_G^3-\omega L+\omega Lv_G^4)+\\
+(v_G+1)^3(-2i+\omega L(v_G+1))].
\end{array} \tag{84}$$

Because it is

$$\Upsilon_l(\omega)=\Upsilon_1(\omega)+\Upsilon_2(\omega), \tag{85}$$

from eqs. (76), (80) and (84) it results that the function

$$\begin{array}{c}
\Upsilon_l(\omega)=(1-v_G^2)\exp[i\omega L(1+v_G)]+\frac{1}{2\omega L(v_G^2-1)^2}[\exp[2i\omega L](v_G+1)^3(-2i+\omega L(v_G-1)+\\
2\exp[i\omega L(1+v_G)](6iv_G+2iv_G^3-\omega L+\omega Lv_G^4)+\\
+(v_G+1)^3(-2i+\omega L(v_G+1))].
\end{array} \tag{86}$$

is the response function of an arm of our interferometer located in the z-axis, due to the longitudinal component of the massive scalar gravitational wave propagating in the same direction of the axis.

For $v_G\rightarrow 1$ it is $\Upsilon_l(\omega)\rightarrow 0$.

The longitudinal response function (86) has been obtained in function of the group velocity of the wave-packet. But, putting eq. (21) into eq. (86) it also results

$$\begin{array}{c}
\Upsilon_l(\omega)=\frac{1}{m^4\omega^2L}(\frac{1}{2}(1+\exp[2i\omega L])m^2\omega^2L(m^2-2\omega^2)+\\
-i\exp[2i\omega L]\omega^2\sqrt{-m^2+\omega^2}(4\omega^2+m^2(-1-iL\omega))+\\
+\omega^2\sqrt{-m^2+\omega^2}(-4i\omega^2+m^2(i+\omega L))+\\
+\exp[iL(\omega+\sqrt{-m^2+\omega^2})](m^6L+m^4\omega^2L+8i\omega^4\sqrt{-m^2+\omega^2}+\\
+m^2(-2L\omega^4-2i\omega^2\sqrt{-m^2+\omega^2}))+2\exp[i\omega L]\omega^3(-3m^2+4\omega^2)\sin[\omega L]).
\end{array} \tag{87}$$

Using the second of equations (10) equation (87) reads

$$\Upsilon_l(\omega) = \frac{1}{(\frac{\beta R_0}{2R_0-12})^4\omega^2 L}(\frac{1}{2}(1+\exp[2i\omega L])\frac{\beta R_0}{2R_0-12}\omega^2 L(\frac{\beta R_0}{2R_0-12}-2\omega^2)+$$

$$-i\exp[2i\omega L]\omega^2\sqrt{-\frac{\beta R_0}{2R_0-12}+\omega^2}(4\omega^2+\frac{\beta R_0}{2R_0-12}(-1-iL\omega))+$$

$$+\omega^2\sqrt{-\frac{\beta R_0}{2R_0-12}+\omega^2}(-4i\omega^2+\frac{\beta R_0}{2R_0-12}(i+\omega L))+$$

$$+\exp[iL(\omega+\sqrt{-\frac{\beta R_0}{2R_0-12}+\omega^2})]((\frac{\beta R_0}{2R_0-12})^3 L+(\frac{\beta R_0}{2R_0-12})^4\omega^2 L+8i\omega^4\sqrt{-\frac{\beta R_0}{2R_0-12}+\omega^2}+$$

$$+\frac{\beta R_0}{2R_0-12}(-2L\omega^4-2i\omega^2\sqrt{-\frac{\beta R_0}{2R_0-12}+\omega^2}))+2\exp[i\omega L]\omega^3(-3\frac{\beta R_0}{2R_0-12}+4\omega^2)\sin[\omega L]). \tag{88}$$

In this way it appears that, the detection of this longitudinal component of the general gravitational waves (29) arising from the action (1) at various frequencies can be connected, in principle, to the Ricci scalar.

In figures 4, 5 and 6 are shown the response functions (86) for an arm of the Virgo interferometer ($L = 3Km$) for $v_G = 0.1$ (non-relativistic case), $v_G = 0.9$ (relativistic case) and $v_G = 0.999$ (ultra-relativistic case). We see that in the non-relativistic case the signal is stronger as it could be expected (for $m \to 0$ we expect$\Upsilon_l(\omega) \to 0$). In figures 7, 8, and 9 the same response functions are shown for the Ligo interferometer ($L = 4Km$).

5 Conclusions

The production and the potential detection with interferometers of a hypotetical massive *scalar* component of gravitational radiation which arises from the R^{-1} theory of gravity has been shown. This agrees with the formal equivalence between high order theories of gravity and scalar tensor gravity that is well known in literature.

First it has been shown that a massive scalar mode of gravitational radiation arises from the particular action of he R^{-1} theory of gravity.

After this, it has been shown that the fact that gravitational waves can have a massive component generates a longitudinal force in addition of the transverse one which is proper of the massless case.

Then, the potential interferometric detection of this longitudinal component has been analyzed and the response of an interferometer has been computed. It has also been shown that this longitudinal response function is directly connected with the Ricci curvature scalar.

In the analysis of the response of the interferometers an analysis parallel to the one seen in [6] and [11] has been used.

As a final remark we emphasize that, an investigation on scalar components of gravitational waves opens to the possibility of using the signals seen from interferometers to understand which is the correct theory of gravitation, while

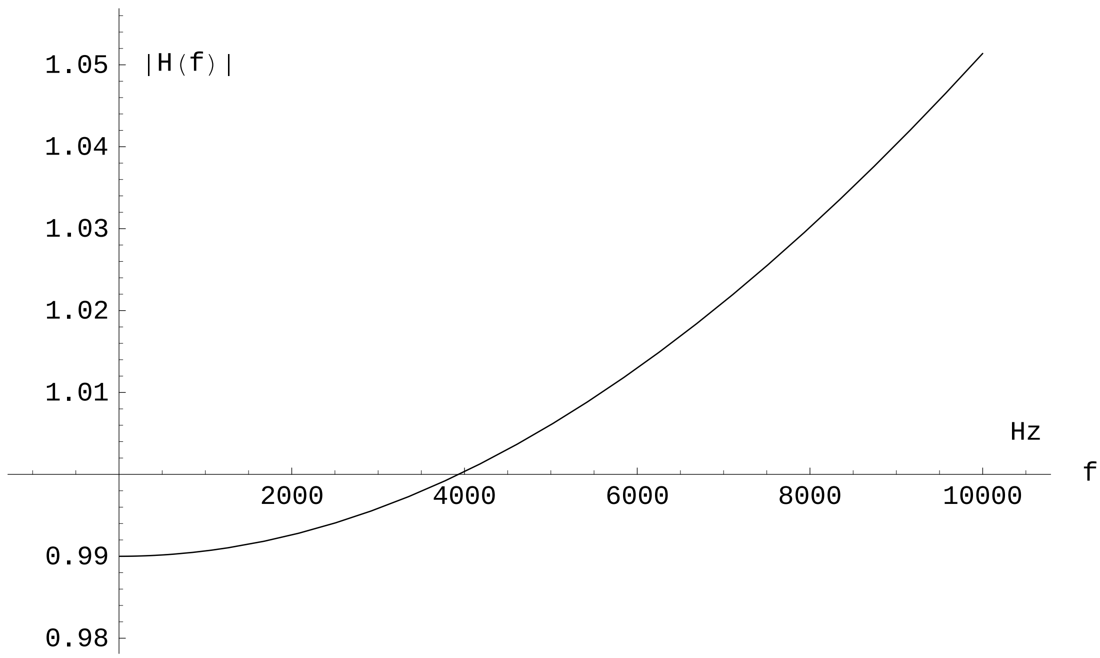

Figure 4: the absolute value of the longitudinal response function (86) of the Virgo interferometer ($L = 3Km$) to a SGW propagating with a speed of $0.1c$ (non relativistic case).

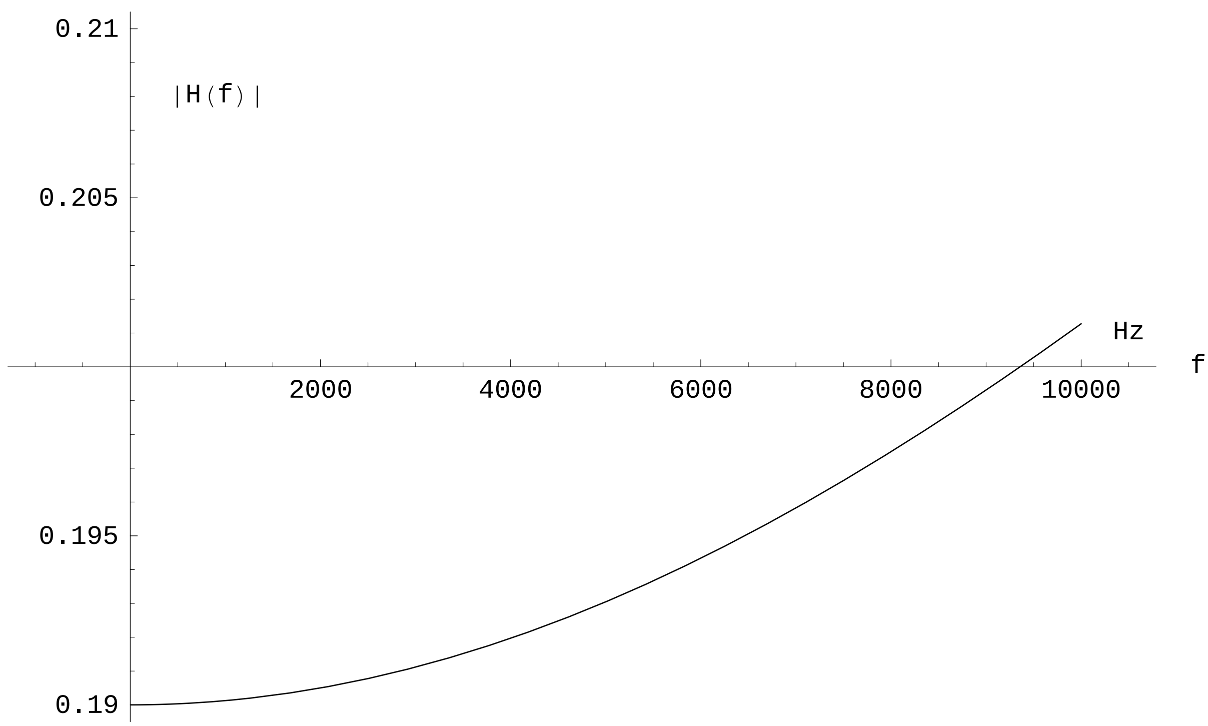

Figure 5: the absolute value of the longitudinal response function (86) of the Virgo interferometer ($L = 3Km$) to a SGW propagating with a speed of 0.9 (relativistic case).

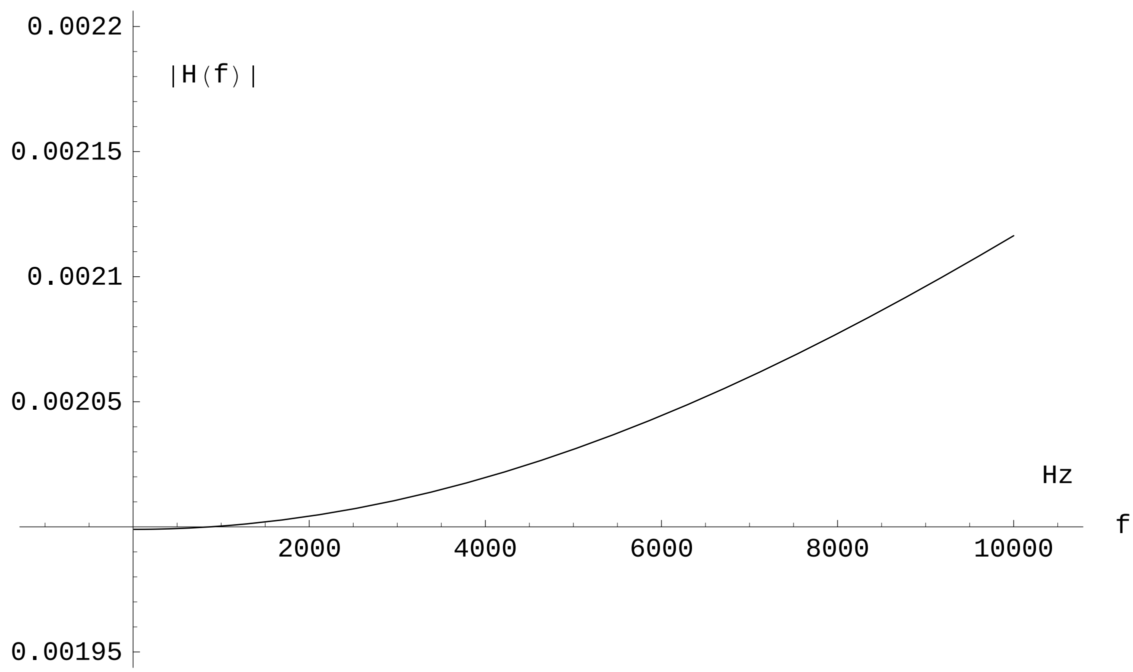

Figure 6: the absolute value of the longitudinal response function (86) of the Virgo interferometer ($L = 3Km$) to a SGW propagating with a speed of 0.999 (ultra relativistic case).

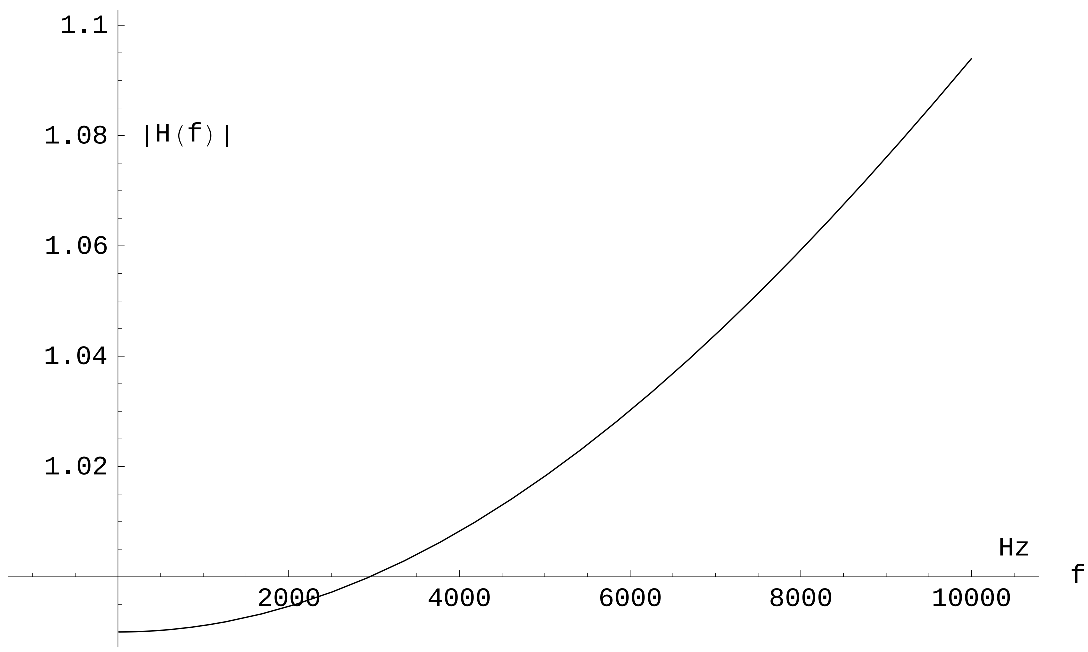

Figure 7: the absolute value of the longitudinal response function (86) of the LIGO interferometer ($L = 4Km$) to a SGW propagating with a speed of $0.1c$ (non relativistic case).

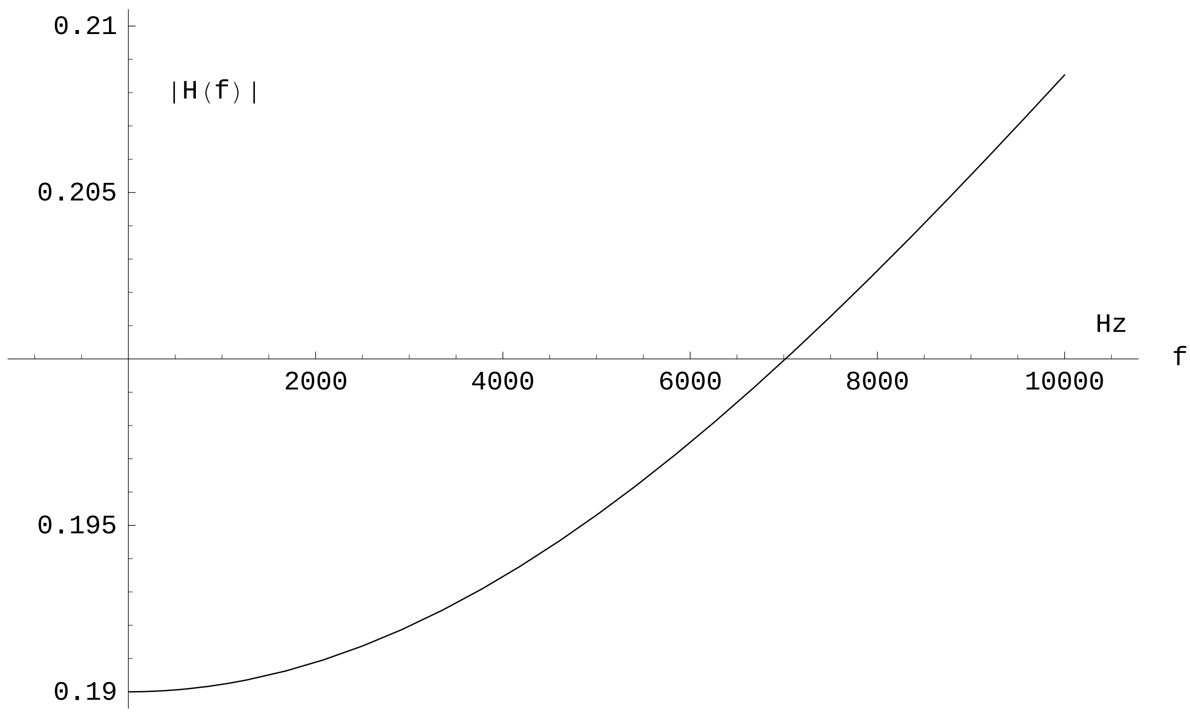

Figure 8: the absolute value of the longitudinal response function (86) of the LIGO interferometer ($L = 4Km$) to a SGW propagating with a speed of $0.9c$ (relativistic case).

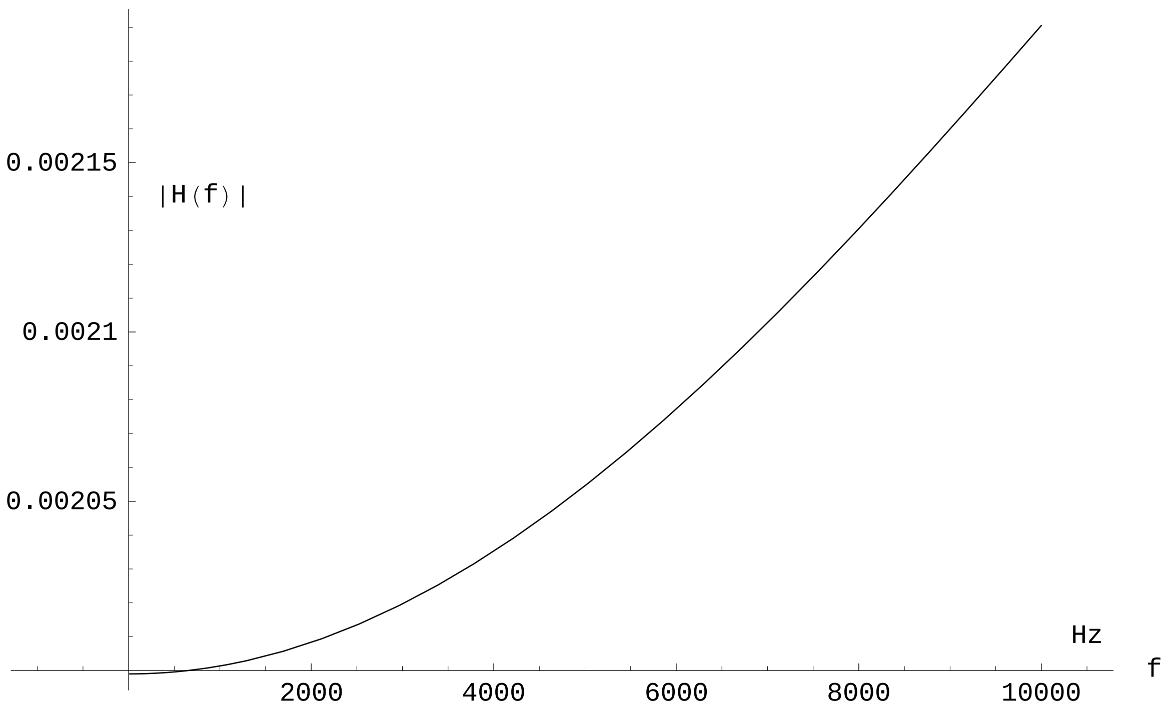

Figure 9: the absolute value of the longitudinal response function (86) of the LIGO interferometer ($L = 4Km$) to a SGW propagating with a speed of $0.999c$ (ultra relativistic case).

the presence of the mass could also have important applications in cosmology because the fact that gravitational waves can have mass could give a contribution to the dark matter of the Universe.

6 Acknowledgements

I would like to thank Salvatore Capozziello, Maria Felicia De Laurentis and Franceso Rubanu for helpful advices during our work. I have to thank the European Gravitational Observatory (EGO) consortium for the using of computing facilities.

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